# Spontaneous emission field of a two-level atom embedded in one-dimensional left-handedand right-handed-material photonic crystals 

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#### Abstract

We investigate the spontaneous emission ( SpE ) of a two-level atom embedded in one-dimensional photonic crystals composed of left-hand material (LHM) and right-hand material. A complete set of mode functions is constructed for quantizing the radiation field. The radiated field distribution under the condition of impedance matching is calculated. The radiated field is focused in each layer and propagates along the direction normal to each layer due to the LHM. With such a structure we can control the propagation of the $\operatorname{SpE}$ field without changing the SpE rate.


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## I. INTRODUCTION

The spontaneous emission $(\mathrm{SpE})$ of an atom is dependent not only on the property of the atom itself, but also on the environment surrounding it. The SpE of an atom placed in a perfect cavity has been investigated for many years [1]. It is found that spontaneous decay can be enhanced or suppressed when the distance between the mirrors varies so that the density of states varies [1]. The SpE rate of an atom in photonic crystals (PC's), in particular for its transition frequency within the photonic band gap (PBG), has been studied in detail [2-4].

In recent years, a new type of material called left-handed material (LHM) [5] has attracted considerable attention. The LHM has a negative refractive index when the permittivity and permeability are negative simultaneously [5]. It was fabricated experimentally [6-8]. In LHM, the wave vector is opposite to the direction of energy propagation. Some unusual phenomena, such as a reverse Doppler shift, reverse Cerenkov radiation, negative refraction, reverse light pressure, and so on, are expected in LHM. Many potential applications-for example, perfect lens which can focus both propagation waves and the evanescent waves-have been predicted [9]. An alternate stack of LHM layers and ordinary dielectric (RHM) layers can construct a new type of PC's with novel properties [10].

The SpE rate of a two-level atom in bulk material of LHM is demonstrated to be $\Gamma=n \mu \Gamma_{0}$ [11], where $\Gamma_{0}$ is the SpE rate of the free space. It is multiple by the permeability $\mu$ to the SpE of ordinary RHM. If a LHM slab is put on a perfect mirror, then there exists a position on another side of the LHM slab from it to the perfect mirror, the optical distance of which is equal to zero. At this position the SpE can be completely inhibited [12]. Since the LHM has quite different refractive behavior from that of the ordinary RHM, the one-dimensional PC (1DPC) of LHM has quite different properties from that of ordinary PC's. It is expected that the SpE rate and the SpE field will be significantly influenced.

In this paper, we constructed a complete set of mode functions for the whole space, including the finite 1DPC structure
and outside space, to quantize the radiation field. We derived the SpE rate for a two-level atom embedded in LHM-RHM 1DPC's. Under the condition of impedance matching ( $I-M$ ) we calculated the distribution of the radiation field. Although the structure does not change the SpE rate, the distribution of the radiation field is significantly changed. The field is focused in each layer and propagating in a directive manner.

## II. MODE FUNCTIONS OF FINITE 1DPC's

The structure of the 1D LHM-RHM PC is shown in Fig. 1. We take the original of the coordinate system at the center of the structure, and the layers are parallel to the $x-y$ plane. A two-level atom is placed in the middle layer, and its position is at $\boldsymbol{r}_{a}\left(0,0, z_{a}\right)$. The atomic dipole moment is equal to $\boldsymbol{p}$. The refractive index, permittivity, permeability, and thickness are indicated by $n_{A}, \varepsilon_{A}, \mu_{A}$, and $d_{A}$ for layer $A$ and $n_{B}, \varepsilon_{B}, \mu_{B}$, and $d_{B}$ for layer $B$, respectively. The layer of the atom located in is taken to be the vacuum, and the thickness is $d_{0}$.

It is known that the complete set of the mode functions in free space is $\left\{\exp \left(i \boldsymbol{K}_{+} \cdot \boldsymbol{r}\right), \exp \left(i \boldsymbol{K}_{-} \cdot \boldsymbol{r}\right)\right\}$, where

$$
\begin{gather*}
K_{ \pm}=\left(K_{x}, K_{y}, \pm K_{z}\right)=K(\sin \theta \cos \phi, \sin \theta \sin \phi, \pm \cos \theta), \\
\quad \theta \in[0, \pi / 2], \phi \in[0,2 \pi], \\
\quad K_{x}, K_{y} \in(-\infty,+\infty), \quad K_{z} \in[0,+\infty) . \tag{1}
\end{gather*}
$$

$\boldsymbol{K}_{+}$and $\boldsymbol{K}_{-}$represent the forward and backward propagations, respectively, and $\left|\boldsymbol{K}_{+}\right|=\left|\boldsymbol{K}_{-}\right|=K$.

When the free space is disturbed by a 1 DPC , the complete set of the mode functions $\left\{\exp \left(i \boldsymbol{K}_{+} \cdot \boldsymbol{r}\right), \exp \left(i \boldsymbol{K}_{-} \cdot \boldsymbol{r}\right)\right\}$ should be replaced by another complete set [13] $\left\{U\left(\boldsymbol{K}_{+}, \lambda, \boldsymbol{r}\right), U\left(\boldsymbol{K}_{-}, \lambda, \boldsymbol{r}\right)\right\}$. According to the scheme of Ref. [13], we express the mode functions together with the unit vectors ( $\hat{\boldsymbol{e}}_{+}, \hat{\boldsymbol{e}}_{-}$) of the field as the following piecewise functions:

$$
U\left(\boldsymbol{K}_{+} K, \lambda, \boldsymbol{r}\right) \hat{\boldsymbol{e}}_{+}= \begin{cases}{\left[e^{i \boldsymbol{K}_{m+} \cdot \boldsymbol{r}} \hat{\boldsymbol{e}}_{m}\left(\boldsymbol{K}_{+}, \lambda\right)+R_{R}^{\lambda} e^{i \boldsymbol{K}_{m-} \cdot \boldsymbol{r}+2 i K_{z} z_{-}-N-1} \hat{\boldsymbol{e}}_{m}\left(\boldsymbol{K}_{-}, \lambda\right)\right],} & z<z_{-N-1}  \tag{2}\\ t_{L m}^{\lambda} e^{i K_{z} z_{-}-N-1}\left[e^{i \boldsymbol{K}_{m+} \cdot \boldsymbol{r}-i K_{m z} z_{m-1}} \hat{\boldsymbol{e}}_{m}\left(\boldsymbol{K}_{+}, \lambda\right)+r_{R m}^{\lambda} e^{i \boldsymbol{K}_{m-} \cdot \boldsymbol{r}+i K_{m z}\left(z_{m-1}+2 d_{m}\right)} \hat{\boldsymbol{e}}_{m}\left(\boldsymbol{K}_{-}, \lambda\right)\right] / D_{m}^{\lambda}, & z_{m-1} \leqslant z<z_{m} \\ T_{L}^{\lambda} e^{i \boldsymbol{K}_{m+} \cdot \boldsymbol{r}+i K_{z}\left(z_{-N-1}-z_{N}\right)} \hat{\boldsymbol{e}}_{m}\left(\boldsymbol{K}_{+}, \lambda\right), & z \geqslant z_{N}\end{cases}
$$

and

$$
U\left(\boldsymbol{K}_{-}, \lambda, \boldsymbol{r}\right) \hat{\boldsymbol{e}}_{-}= \begin{cases}{\left[e^{i \boldsymbol{K}_{m-} \cdot \boldsymbol{r}} \hat{\boldsymbol{e}}_{m}\left(\boldsymbol{K}_{-}, \lambda\right)+R_{L}^{\lambda} e^{i \boldsymbol{K}_{m+} \cdot \boldsymbol{r}-2 i K_{z} z_{N}} \hat{\boldsymbol{e}}_{m}\left(\boldsymbol{K}_{+}, \lambda\right)\right],} & z \geqslant z_{N}  \tag{3}\\ t_{R m}^{\lambda} e^{-i K_{Z} z_{N}}\left[e^{i \boldsymbol{K}_{m-}-\boldsymbol{r}+i K_{m z} z_{m}} \hat{\boldsymbol{e}}_{m}\left(\boldsymbol{K}_{-}, \lambda\right)+r_{L m}^{\lambda} e^{i \boldsymbol{K}_{m+} \cdot \boldsymbol{r}+i K_{m z}\left(2 d_{m}-z_{m}\right)} \hat{\boldsymbol{e}}_{m}\left(\boldsymbol{K}_{+}, \lambda\right)\right] / D_{m}^{\lambda}, & z_{m-1} \leqslant z<z_{m} \\ \left.T_{R}^{\lambda} e^{i \boldsymbol{K}_{m-} \cdot \boldsymbol{r}+i K_{z}\left(z_{-N-1}-z_{N}\right.}\right) \hat{\boldsymbol{e}}_{m}\left(\boldsymbol{K}_{-}, \lambda\right), & z<z_{-N-1}\end{cases}
$$

where

$$
\begin{align*}
& \boldsymbol{K}_{m \pm}=\left(K_{m x}, K_{m y}, \pm K_{m z}\right) \\
&= K_{m}\left(\sin \theta_{m} \cos \phi, \sin \theta_{m} \sin \phi, \pm \cos \theta_{m}\right) \\
&= K\left(\sin \theta \cos \phi, \sin \theta \sin \phi, \pm n_{m} \cos \theta_{m}\right) \\
& \quad K_{m}=n_{m} K \tag{4}
\end{align*}
$$

and the angle $\theta_{m}$ relates to the angle $\theta$ in vacuum according to Snell's law

$$
\begin{equation*}
\sin \theta=n_{m} \sin \theta_{m} \tag{5}
\end{equation*}
$$

The superscript $\lambda=1,2$ indicates two transverse polarization directions. The unit vectors of the two perpendicular polarizations are

$$
\begin{gather*}
\hat{\boldsymbol{e}}_{m}\left(\boldsymbol{K}_{ \pm}, \lambda=1\right)=(\sin \phi,-\cos \phi, 0), \\
\hat{\boldsymbol{e}}_{m}\left(\boldsymbol{K}_{ \pm}, \lambda=2\right)=\left(\cos \theta_{m} \cos \phi, \cos \theta_{m} \sin \phi, \mp \sin \theta_{m}\right) \tag{6}
\end{gather*}
$$

They are determined by the direction of the wave vector in the $m$ th layer. In the expressions of mode functions (2) and (3), $t_{L m}^{\lambda}\left(t_{R m}^{\lambda}\right)$ denotes the transmission coefficient through the left (right) part of the $m$ th layer $\left(z_{m-1}<z<z_{m}\right), r_{R m}^{\lambda}\left(r_{L m}^{\lambda}\right)$ denotes the reflective coefficient on the right (left) interface of the $m$ th layer, and $T_{R}^{\lambda}\left(T_{L}^{\lambda}\right)$ denotes the total transmission
coefficient of PC coming from the right (left) interface of the region of $z<z_{-N-1}\left(z>z_{N}\right) . R_{R}^{\lambda}\left(R_{L}^{\lambda}\right)$ denotes the total reflective coefficient on the right (left) interface of the region of $z<z_{-N-1}\left(z>z_{N}\right)$. Notice that these entire coefficients can be expanded to the recurrent formations of the reflection and transmission coefficient at each interface. $D_{m}^{\lambda}$ originates from the multireflection effect in the $m$ th layer,

$$
\begin{equation*}
D_{m}^{\lambda}=1-r_{L m}^{\lambda} r_{R m}^{\lambda} e^{2 i K_{m z} d_{m}}, \tag{7}
\end{equation*}
$$

where $r_{L m}^{\lambda}$ and $r_{R m}^{\lambda}$ are the effective reflection coefficients on the multilayered structure (not a single interface) and will be determined recurrently. The multireflected effect on the other interfaces is contained in the calculation of $r_{L m}^{\lambda}$ and $r_{R m}^{\lambda}$.

The set of $\left\{U\left(\boldsymbol{K}_{+}, \lambda, \boldsymbol{r}\right), U\left(\boldsymbol{K}_{-}, \lambda, \boldsymbol{r}\right)\right\}$ contains all modes of the whole space including the region of the 1DPC and the outsides. The orthogonal relation and completeness relation of the modes can be expressed with the notation of Eqs. (2) and (3) as

$$
\begin{gather*}
\int d \boldsymbol{r}\left[U\left(\boldsymbol{K}_{+}, \lambda, \boldsymbol{r}\right) \hat{\boldsymbol{e}}_{+}\right] \cdot\left[U\left(\boldsymbol{K}_{-}^{\prime}, \lambda^{\prime}, \boldsymbol{r}\right) \hat{\boldsymbol{e}}_{-}\right]=0  \tag{8}\\
\int d \boldsymbol{r}\left[U\left(\boldsymbol{K}_{ \pm}, \lambda, \boldsymbol{r}\right) \hat{\boldsymbol{e}}_{ \pm}\right] \cdot\left[U\left(\boldsymbol{K}_{ \pm}^{\prime}, \lambda^{\prime}, \boldsymbol{r}\right) \hat{\boldsymbol{e}}_{ \pm}\right] \\
\quad=(2 \pi)^{3} \delta_{\lambda \lambda^{\prime}} \delta\left(\boldsymbol{K}_{ \pm}-\boldsymbol{K}_{ \pm}^{\prime}\right) . \tag{9}
\end{gather*}
$$



## III. SPONTANEOUS EMISSION DECAY

The quantized electric field operator can be expressed as

$$
\begin{equation*}
\boldsymbol{E}(\boldsymbol{r}, t)=\boldsymbol{E}^{(+)}(\boldsymbol{r}, t)+\boldsymbol{E}^{(-)}(\boldsymbol{r}, t) . \tag{10}
\end{equation*}
$$

In free space, we have

$$
\begin{gather*}
\boldsymbol{E}^{(+)}(\boldsymbol{r}, t)=\sum_{\boldsymbol{K}, \lambda} \hat{\boldsymbol{e}}_{0}(\boldsymbol{K}, \lambda) \varepsilon_{K}^{0} e^{i \boldsymbol{K} \cdot \boldsymbol{r}} a_{\boldsymbol{K}} e^{-i \nu_{K} t}, \\
\boldsymbol{E}^{(-)}(\boldsymbol{r}, t)=\left[\boldsymbol{E}^{(+)}(\boldsymbol{r}, t)\right]^{*}, \tag{11}
\end{gather*}
$$

where $\varepsilon_{K}^{0}=\left(\hbar \nu_{K} / 2 \varepsilon_{0} V\right)^{1 / 2}$, the unit vector $\hat{\boldsymbol{e}}_{0}(\boldsymbol{K}, \lambda)$ indicates the polarization direction, and $\nu_{K}=c K$ is the angular frequency of the field. In the interaction picture, the interaction Hamiltonian between a two-level atom and field can be written as

$$
\begin{equation*}
V_{I}(t)=\hbar \sum_{\boldsymbol{K} \lambda}\left[g_{\boldsymbol{K}}^{\lambda}\left(\boldsymbol{r}_{a}\right) \sigma_{+} a_{\boldsymbol{K} \lambda} e^{i\left(\omega_{0}-\nu_{K}\right) t}+\text { H.c. }\right], \tag{12}
\end{equation*}
$$

where $\omega_{0}$ is the atomic transition frequency, $\sigma_{+}=\left(\sigma_{-}\right)^{+}$is the atomic transition operator, $\boldsymbol{r}_{a}$ indicates the position of the atom, and

$$
\begin{equation*}
g_{\boldsymbol{K}}^{\lambda}\left(\boldsymbol{r}_{a}\right)=-\frac{\boldsymbol{p} \cdot \hat{\boldsymbol{e}}_{0}(\boldsymbol{K}, \lambda) \varepsilon_{K}^{0}}{\hbar} e^{i \boldsymbol{K} \cdot \boldsymbol{r}_{a}} \tag{13}
\end{equation*}
$$

is the atom-field coupling coefficient, where $\boldsymbol{p}$ is the atomic dipole moment.

In the present of 1DPC structure, the mode functions of plane wave should be replaced by $\left\{U\left(\boldsymbol{K}_{+}, \lambda, \boldsymbol{r}\right), U\left(\boldsymbol{K}_{-}, \lambda, \boldsymbol{r}\right)\right\}$. The positive-frequency part of electric field operator for our system is modified as

$$
\begin{align*}
\boldsymbol{E}^{(+)}(\boldsymbol{r}, t)= & \sum_{\boldsymbol{K}_{+}, \lambda} U\left(\boldsymbol{K}_{+}, \lambda, \boldsymbol{r}\right) \hat{\boldsymbol{e}}_{+} \varepsilon_{K}^{m} a_{\boldsymbol{K}_{+} \lambda} e^{-i \nu_{K} t} \\
& +\sum_{\boldsymbol{K}_{-}, \lambda} U\left(\boldsymbol{K}_{-}, \lambda, \boldsymbol{r}\right) \hat{\boldsymbol{e}}_{-} \varepsilon_{K}^{m} a_{\boldsymbol{K}_{-} \lambda} e^{-i \nu_{K^{\prime}} t} \tag{14}
\end{align*}
$$

where $\nu_{K}=c\left|\boldsymbol{K}_{ \pm}\right|=c K, \varepsilon_{K}^{m}=\left(\hbar \nu_{K} / 2\left|\varepsilon_{m}\right| V\right)^{1 / 2}$ with $m$ indicating the $m$ th layer where the position $r$ is in, and $\varepsilon_{m}$ is the dielectric constant of the $m$ th layer. The interaction Hamiltonian is expressed as

$$
\begin{align*}
V_{I}(t)= & \hbar \sum_{\boldsymbol{K}_{+}, \lambda}\left[g_{\boldsymbol{K}_{+}}^{\lambda}\left(\boldsymbol{r}_{a}\right) \sigma_{+} a_{\boldsymbol{K}_{+} \lambda} e^{i\left(\omega_{0}-\nu_{K}\right) t}+\text { H.c. }\right] \\
& +\hbar \sum_{\boldsymbol{K}_{-}, \lambda}\left[g_{\boldsymbol{K}_{-}}^{\lambda}\left(\boldsymbol{r}_{a}\right) \sigma_{+} a_{\boldsymbol{K}_{-} \lambda} e^{i\left(\omega_{0}-\nu_{K}\right) t}+\text { H.c. }\right] . \tag{15}
\end{align*}
$$

Notice that layer 0 where the atom is located in is vacuum, so that the expression of wave vector in layer 0 can be replaced by the notation in vacuum as in Eq. (1). With the help of the second terms in Eqs. (2) and (3), the atom-field coupling coefficients $g_{\boldsymbol{K}_{ \pm}}^{\lambda}\left(\boldsymbol{r}_{a}\right)$ can be obtained as

$$
\begin{align*}
g_{\boldsymbol{K}_{+}}^{\lambda}\left(\boldsymbol{r}_{a}\right)= & -\frac{\varepsilon_{K}^{0}}{\hbar}\left(t_{L 0}^{\lambda} / D_{0}^{\lambda}\right) e^{i \boldsymbol{K}_{+} \cdot \boldsymbol{r}_{a}+i K\left(z_{-N-1}+d_{0} / 2\right) \cos \theta}\left[\boldsymbol{p} \cdot \hat{\boldsymbol{e}}_{0}\left(\boldsymbol{K}_{+}, \lambda\right)\right. \\
& \left.+\boldsymbol{p} \cdot \hat{\boldsymbol{e}}_{0}\left(\boldsymbol{K}_{-}, \lambda\right) r_{R 0}^{\lambda} e^{-i K\left(2 z_{a}-d_{0}\right) \cos \theta}\right], \tag{16}
\end{align*}
$$

$$
\begin{align*}
g_{\boldsymbol{K}_{-}}^{\lambda}\left(\boldsymbol{r}_{a}\right)= & -\frac{\varepsilon_{K}^{0}}{\hbar}\left(t_{R 0}^{\lambda} / D_{0}^{\lambda}\right) e^{i \boldsymbol{K}_{-} \cdot \boldsymbol{r}_{a}+i K\left(d_{0} / 2-z_{N}\right) \cos \theta}\left[\boldsymbol{p} \cdot \hat{\boldsymbol{e}}_{0}\left(\boldsymbol{K}_{-}, \lambda\right)\right. \\
& \left.+\boldsymbol{p} \cdot \hat{\boldsymbol{e}}_{0}\left(\boldsymbol{K}_{+}, \lambda\right) r_{L 0}^{\lambda} e^{i K\left(2 z_{a}+d_{0}\right) \cos \theta}\right] \tag{17}
\end{align*}
$$

and the position of the atom is at

$$
\begin{equation*}
\boldsymbol{r}_{a}=\left(0,0, z_{a}\right) \quad \text { where } z_{-1}<z_{a}<z_{0} . \tag{18}
\end{equation*}
$$

The state vector of the system can be expressed as

$$
\begin{align*}
\left|\psi_{I}(t)\right\rangle= & C_{a}(t)|a, 0\rangle+\sum_{\boldsymbol{K}_{+}, \lambda} C_{b \boldsymbol{K}_{+} \lambda}(t)\left|b, 1_{\boldsymbol{K}_{+} \lambda}\right\rangle \\
& +\sum_{\boldsymbol{K}_{-}, \lambda} C_{b \boldsymbol{K}_{-} \lambda}(t)\left|b, 1_{\boldsymbol{K}_{-}}\right\rangle . \tag{19}
\end{align*}
$$

The state $|a, 0\rangle$ is for the atom in the excited state and no photon, and $\left|b, 1_{K_{ \pm} \lambda}\right\rangle$ is for the atom in the ground state with a photon of $\left(\boldsymbol{K}_{ \pm}, \stackrel{\lambda}{ }\right)$. We assume the initial state is that the atom is in the excited state and there is no photon-i.e., $C_{a}(0)=1$ and $C_{b K_{ \pm}}(0)=0$. From the Schrödinger equation in the interaction picture,

$$
\begin{equation*}
\frac{\partial}{\partial t}\left|\psi_{I}(t)\right\rangle=-\frac{i}{\hbar} V_{I}\left|\psi_{I}(t)\right\rangle, \tag{20}
\end{equation*}
$$

we obtain the atomic dynamical equations

$$
\begin{align*}
& \frac{d}{d t} C_{a}(t)=-i \sum_{\boldsymbol{K}_{+}, \lambda} g_{\boldsymbol{K}_{+}}^{\lambda}\left(\boldsymbol{r}_{a}\right) C_{b \boldsymbol{K}_{+} \lambda}(t) e^{i\left(\omega_{0}-\nu_{K}\right) t} \\
&-i \sum_{\boldsymbol{K}_{-}, \lambda} g_{\boldsymbol{K}_{-}}^{\lambda}\left(\boldsymbol{r}_{a}\right) C_{b \boldsymbol{K}_{-} \lambda}(t) e^{i\left(\omega_{0}-\nu_{K}\right) t},  \tag{21}\\
& \frac{d}{d t} C_{b \boldsymbol{K}_{+} \lambda}(t)=-i\left[g_{\boldsymbol{K}_{+}}^{\lambda}\left(\boldsymbol{r}_{a}\right)\right]^{*} e^{-i\left(\omega_{0}-\nu_{K}\right) t} C_{a}(t),  \tag{22}\\
& \frac{d}{d t} C_{b \boldsymbol{K}_{-} \lambda}(t)=-i\left[g_{\boldsymbol{K}_{-}}^{\lambda}\left(\boldsymbol{r}_{a}\right)\right]^{*} e^{-i\left(\omega_{0}-\nu_{K}\right) t} C_{a}(t) . \tag{23}
\end{align*}
$$

Integrating Eqs. (22) and (23) over time, then substituting them into Eq. (21), with the help of a transformation from summarization over $\boldsymbol{K}_{+}$and $\boldsymbol{K}_{-}$into integrals:
$\sum_{\boldsymbol{K}_{ \pm}} \rightarrow \frac{V}{(2 \pi)^{3}} \int d \boldsymbol{K}_{ \pm}=\frac{V}{(2 \pi)^{3}} \int_{0}^{\infty} d K \int_{0}^{\pi / 2} d \theta \int_{0}^{2 \pi} d \phi K^{2} \sin \theta$.

Then Eq. (21) becomes

$$
\begin{align*}
\frac{d}{d t} C_{a}(t)= & -\frac{V}{(2 \pi)^{3}} \int_{0}^{2 \pi} d \phi \int_{0}^{\pi / 2} d \theta \sin \theta \int_{0}^{\infty} d K K^{2} \int_{0}^{t} d t^{\prime} \\
& \times \sum_{\lambda=1}^{2}\left[\left|g_{\boldsymbol{K}_{+}}^{\lambda}\left(\boldsymbol{r}_{a}\right)\right|^{2}+\left|g_{\boldsymbol{K}_{-}}^{\lambda}\left(\boldsymbol{r}_{a}\right)\right|^{2}\right] C_{a}\left(t^{\prime}\right) e^{i c(k-K)\left(t-t^{\prime}\right)}, \tag{25}
\end{align*}
$$

where $k=\omega_{0} / c$ and $K=\nu_{K} / c$.
With the Weisskopf-Wigner approximation, we assume that the dependence of $K^{2} \Sigma_{\lambda=1}^{2}\left[\left|g_{\boldsymbol{K}_{+}}^{\lambda}\left(\boldsymbol{r}_{a}\right)\right|^{2}+\left|g_{\boldsymbol{K}_{-}}^{\lambda}\left(\boldsymbol{r}_{a}\right)\right|^{2}\right]$ on $K$ can simply be replaced by $k$, so it can be taken out of the
integration over $K$. We extend the lower limit in $K$ integration to $-\infty$ and obtain from Eq. (25) that

$$
\begin{align*}
\frac{d}{d t} C_{a}(t) \approx & -\frac{V k^{2}}{(2 \pi)^{3}} \int_{0}^{2 \pi} d \phi \int_{0}^{\pi / 2} d \theta \sin \theta \sum_{\lambda=1}^{2}\left[\left|g_{k_{+}}^{\lambda}\left(\mathbf{r}_{a}\right)\right|^{2}\right. \\
& \left.+\left|g_{k_{-}}^{\lambda}\left(\mathbf{r}_{a}\right)\right|^{2}\right] \int_{0}^{t} d t^{\prime} C_{a}\left(t^{\prime}\right) \int_{-\infty}^{\infty} d K e^{i c(k-K)\left(t-t^{\prime}\right)} \\
= & -\frac{V k^{2}}{(2 \pi)^{2} c} \int_{0}^{2 \pi} d \phi \int_{0}^{\pi / 2} d \theta \sin \theta \sum_{\lambda=1}^{2}\left[\left|g_{k_{+}}^{\lambda}\left(\mathbf{r}_{a}\right)\right|^{2}\right. \\
& \left.+\left|g_{k_{-}}^{\lambda}\left(\mathbf{r}_{a}\right)\right|^{2}\right] \int_{0}^{t} d t^{\prime} C_{a}\left(t^{\prime}\right) \delta\left(t-t^{\prime}\right) \\
= & -\frac{V k^{2}}{2(2 \pi)^{2} c} \int_{0}^{2 \pi} d \phi \int_{0}^{\pi / 2} d \theta \sin \theta \sum_{\lambda=1}^{2}\left[\left|g_{k_{+}}^{\lambda}\left(\mathbf{r}_{a}\right)\right|^{2}\right. \\
& \left.+\left|g_{k_{-}}^{\lambda}\left(\mathbf{r}_{a}\right)\right|^{2}\right] C_{a}(t) \\
= & -\frac{1}{2} \Gamma C_{a}(t), \tag{26}
\end{align*}
$$

where

$$
\begin{equation*}
\Gamma=\frac{V k^{2}}{(2 \pi)^{2} c} \int_{0}^{2 \pi} d \phi \int_{0}^{\pi / 2} d \theta \sin \theta \sum_{\lambda=1}^{2}\left[\left|g_{k_{+}}^{\lambda}\left(\boldsymbol{r}_{a}\right)\right|^{2}+\left|g_{k_{-}}^{\lambda}\left(\boldsymbol{r}_{a}\right)\right|^{2}\right] . \tag{27}
\end{equation*}
$$

As we know, the spontaneous emission of the excited two-level atom is related to the environment surrounding it. In our method, the environment is presented by the reflection and transmission coefficients. The atomic evolution in the 1 DPC is quite different from that in the vacuum due to the reflected field.

However, under the condition of $I-M$ the reflection coefficient is equal to zero and then the radiated field can be calculated easily. Since we focus on the effect of LHM on the radiation field in this paper, we consider only the situation of $I-M$, which could make the effect of LHM clear without the complexity caused by multilayer reflection.

For $\varepsilon_{A}=-\varepsilon_{B}, \mu_{A}=-\mu_{B}$ and $n_{A}=\sqrt{\varepsilon_{A}} \sqrt{\mu_{A}}=1, n_{B}=\sqrt{\varepsilon_{B}} \sqrt{\mu_{B}}$ $=-1$, it satisfies the condition of $I-M$, so that all reflection coefficients are equal to zero. Equations (16) and (17) reduce to

$$
\begin{equation*}
g_{\boldsymbol{K}_{+}}^{\lambda}\left(\boldsymbol{r}_{a}\right)=-\frac{\boldsymbol{\varepsilon}_{K}^{0}}{\hbar} t_{L 0}^{\lambda} e^{i K\left(z_{a}+d_{0} / 2+z_{-N-1}\right) \cos \theta} \boldsymbol{p} \cdot \hat{\boldsymbol{e}}_{0}\left(\boldsymbol{K}_{+}, \lambda\right), \tag{28}
\end{equation*}
$$

$$
\begin{equation*}
g_{\boldsymbol{K}_{-}}^{\lambda}\left(\boldsymbol{r}_{a}\right)=-\frac{u\left(\boldsymbol{r}_{N}^{a}\right) \varepsilon_{K}^{0}}{\hbar} t_{R 0}^{\lambda} e^{-i K\left(z_{a}-d_{0} / 2+z_{N}\right) \cos } \theta_{\boldsymbol{p}} \cdot \hat{\boldsymbol{e}}_{0}\left(\boldsymbol{K}_{-}, \lambda\right) \tag{29}
\end{equation*}
$$

and the transmission coefficients become a phase shift factor-that is,

$$
\begin{equation*}
t_{R m}^{\lambda}=\exp \left(i K \sum_{l=m+1}^{N} n_{l} d_{l} \cos \theta_{l}\right), \tag{30}
\end{equation*}
$$

$$
\begin{equation*}
t_{L m}^{\lambda}=\exp \left(i K \sum_{l=-N}^{m-1} n_{l} d_{l} \cos \theta_{l}\right) \tag{31}
\end{equation*}
$$

hence

$$
\begin{gather*}
\left(t_{L 0}^{\lambda}\right)^{*} t_{L m}^{\lambda}=\exp \left(i K \cos \theta \sum_{l=0}^{m-1} n_{l} d_{l}\right),  \tag{32}\\
\left(t_{R 0}^{\lambda}\right)^{*} t_{R m}^{\lambda}=\exp \left(-i K \cos \theta \sum_{l=1}^{m-1} n_{l} d_{l}\right) . \tag{33}
\end{gather*}
$$

Consequently, inserting Eqs. (28)-(33) into Eq. (27) and after some simple deductions, we can find that, in such a case, the atomic decay rate is equal to the decay rate in vacuumi.e., $\Gamma=\Gamma_{0}$. So we get

$$
\begin{equation*}
\frac{d}{d t} C_{a}(t)=-\frac{\Gamma_{0}}{2} C_{a}(t) \tag{34}
\end{equation*}
$$

where $\Gamma_{0}=p^{2} k^{3} / 3 \varepsilon_{0} \pi \hbar$ is the decay rate in the vacuum. Although the dynamic of the SpE decay is the same as that in the vacuum under the condition of $I-M$, we will see in the next section that the field of the SpE is completely changed in the LHM-RHM 1DPC.

## IV. DISTRIBUTION OF SPONTANEOUS EMISSION FIELD

In the 1 DPC , the calculation of the radiated field is very complicated because of the multireflection, particularly in the early stage of the decay when the multireflected field has not approached the stationary state. However, under the condition of $I-M$, as mentioned above, the dynamic of SpE decay becomes the same as in the vacuum. Equation (34) gives $C_{a}(t)=e^{-\Gamma_{0} t / 2}$. Substituting it into Eqs. (22) and (23) and integrating over time we get

$$
\begin{align*}
& C_{b \boldsymbol{K}_{+} \lambda}(t)=-i \int_{0}^{t}\left[g_{\boldsymbol{K}_{+}}^{\lambda}\left(\boldsymbol{r}_{a}\right)\right]^{*} e^{-i\left(\omega_{0}-\nu_{K}\right) t^{\prime}-\left(\Gamma_{0} / 2\right) t^{\prime}} d t^{\prime},  \tag{35}\\
& C_{b \boldsymbol{K}_{-} \lambda}(t)=-i \int_{0}^{t}\left[g_{\boldsymbol{K}_{-}}^{\lambda}\left(\boldsymbol{r}_{a}\right)\right]^{*} e^{-i\left(\omega_{0}-\nu_{K}\right) t^{\prime}-\left(\Gamma_{0} / 2\right) t^{\prime}} d t^{\prime}, \tag{36}
\end{align*}
$$

and substituting them into Eq. (19) we have

$$
\begin{equation*}
\left|\psi_{I}(t)\right\rangle=e^{-\Gamma_{0} t / 2}|a, 0\rangle+|b\rangle\left|\gamma_{0}(t)\right\rangle, \tag{37}
\end{equation*}
$$

where

$$
\begin{align*}
\left|\gamma_{0}(t)\right\rangle= & -i \int_{0}^{t}\left[g_{\boldsymbol{K}_{+}}^{\lambda}\left(\boldsymbol{r}_{a}\right)\right]^{*} e^{-i\left(\omega_{0}-\nu_{K}\right) t^{\prime}-\left(\Gamma_{0} / 2\right) t^{\prime}} d t^{\prime}\left|1_{\boldsymbol{K}_{+} \lambda}\right\rangle \\
& -i \int_{0}^{t}\left[g_{\boldsymbol{K}_{-}}^{\lambda}\left(\boldsymbol{r}_{a}\right)\right]^{*} e^{-i\left(\omega_{0}-\nu_{K}\right) t^{\prime}-\left(\Gamma_{0} / 2\right) t^{\prime}} d t^{\prime}\left|1_{\boldsymbol{K}_{-} \lambda}\right\rangle \tag{38}
\end{align*}
$$

represents the state of the radiated field.
The radiated field is given by the first-order correlation function

$$
\begin{align*}
G^{(1)}(\boldsymbol{r}, \boldsymbol{r} ; t, t) & =\left\langle\psi_{I}(t)\right| \boldsymbol{E}^{(-)}(\boldsymbol{r}, t) \boldsymbol{E}^{(+)}(\boldsymbol{r}, t)\left|\psi_{I}(t)\right\rangle \\
& \left.=\left|\langle 0| \boldsymbol{E}^{(+)}(\boldsymbol{r}, t)\right| \gamma_{0}(t)\right\rangle\left.\right|^{2} . \tag{39}
\end{align*}
$$

When the reflection coefficients are equal to zero, the field operator of Eq. (14) can be written as

$$
\begin{align*}
\boldsymbol{E}^{(+)}(\boldsymbol{r}, t)= & \sum_{\boldsymbol{K}_{+}, \lambda} \varepsilon_{K}^{m} t_{L m}^{\lambda} \hat{\boldsymbol{e}}_{m}\left(\boldsymbol{K}_{+}, \lambda\right) e^{i \boldsymbol{K}_{m+} \cdot \boldsymbol{r}-i K_{m z} z_{m-1}+i K_{z} z_{-N-1}} a_{\boldsymbol{K}_{+}, \lambda} e^{-i \nu_{k} t} \\
& +\sum_{\boldsymbol{K}_{-}, \lambda} \varepsilon_{K}^{m} t_{R m}^{\lambda} \hat{\boldsymbol{e}}_{m}\left(\boldsymbol{K}_{-}, \lambda\right) e^{i \boldsymbol{K}_{m-} \cdot \boldsymbol{r}+i K_{m z} z_{m}-i K_{z} z_{N}} a_{\boldsymbol{K}_{-}, \lambda} e^{-i \nu_{k} t}, \tag{40}
\end{align*}
$$

where the subscript $m$ is determined by the layer where $\boldsymbol{r}$ is located in. Substituting Eqs. (28) and (29) into Eq. (40) we get

$$
\begin{align*}
&\langle 0| \boldsymbol{E}^{(+)}(r, t)\left|\gamma_{0}(t)\right\rangle \\
&= i \frac{V \varepsilon_{K}^{m} \varepsilon_{K}^{0}}{\hbar(2 \pi)^{3}} \sum_{\lambda} \int_{0}^{\pi / 2} d \theta \sin \theta \int_{0}^{2 \pi} d \boldsymbol{\phi} \int_{0}^{\infty} d K K^{2} \int_{0}^{t} d t^{\prime} \\
& \times e^{-i\left(\omega_{0}-\nu_{K}\right) t^{\prime}-\left(\Gamma_{0} / 2\right) t^{\prime}} e^{-i \nu_{K^{t}}} \\
& \times\left\{\exp \left(i K \cdot \mathcal{P}_{+}\right) \boldsymbol{p} \cdot \hat{\boldsymbol{e}}_{0}\left(\boldsymbol{K}_{+}, \lambda\right) \hat{\boldsymbol{e}}_{m}\left(\boldsymbol{K}_{+}, \lambda\right)\right. \\
&\left.+\exp \left(i K \cdot \mathcal{P}_{-}\right) \boldsymbol{p} \cdot \hat{\boldsymbol{e}}_{0}\left(\boldsymbol{K}_{-}, \lambda\right) \hat{\boldsymbol{e}}_{m}\left(\boldsymbol{K}_{-}, \lambda\right)\right\} \tag{41}
\end{align*}
$$

Here we have defined two effective paths $\mathcal{P}_{+}$and $\mathcal{P}_{-}$for the forward and backward waves, respectively. Assume the atom is located at the original of the coordinate system. The two effective paths can be expressed as

$$
\begin{equation*}
\mathcal{P}_{+}(x, y, z, \theta, \phi)=x \sin \theta \cos \phi+y \sin \theta \sin \phi+\widetilde{z}_{m} \cos \theta, \tag{42}
\end{equation*}
$$

$$
\begin{equation*}
\mathcal{P}_{-}(x, y, z, \theta, \phi)=x \sin \theta \cos \phi+y \sin \theta \sin \phi-\widetilde{z}_{m} \cos \theta, \tag{43}
\end{equation*}
$$

where

$$
\begin{equation*}
\tilde{z}_{m}(z)=\frac{d_{0}}{2}+\sum_{l=1}^{m-1} n_{l} d_{l}+n_{m}\left(z-z_{m-1}\right) . \tag{44}
\end{equation*}
$$

If we let $\theta=\pi-\theta^{\prime}$ in the second term in the integral (41), we find that the integrals of the two terms relating to $\boldsymbol{K}_{+}$and $\boldsymbol{K}_{-}$ can be combined into one with $\theta$ varying from 0 to $\pi$. Thus, the field becomes

$$
\begin{align*}
\langle 0| \boldsymbol{E}^{(+)}(\boldsymbol{r}, t)\left|\gamma_{0}(t)\right\rangle= & i \frac{V \varepsilon_{K}^{m} \varepsilon_{K}^{0}}{\hbar(2 \pi)^{3}} \int_{0}^{\pi} d \theta \sin \theta \int_{0}^{2 \pi} d \phi \int_{0}^{\infty} d K K^{2} \\
& \times \int_{0}^{t} d t^{\prime} e^{-i\left(\omega_{0}-\nu_{K}\right) t^{\prime}-\left(\Gamma_{0} / 2\right) t^{\prime}} e^{-i \nu_{K^{t}}} e^{i \boldsymbol{K} \cdot \tilde{r}} \boldsymbol{p} \cdot \stackrel{\leftrightarrow}{U}_{m}, \tag{45}
\end{align*}
$$

where

$$
\begin{equation*}
\widetilde{\boldsymbol{r}}=\left(x, y, \widetilde{z}_{m}(z)\right) \tag{46}
\end{equation*}
$$

and

$$
\begin{gather*}
\boldsymbol{K}=K(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta), \\
0 \leqslant \theta \leqslant \pi, \quad 0 \leqslant \phi \leqslant 2 \pi . \tag{47}
\end{gather*}
$$

In Eq. (48) we define a tensor $\stackrel{\leftrightarrow}{U}_{m}$ as

$$
\begin{equation*}
\stackrel{\leftrightarrow}{U}_{m}=\sum_{\lambda=1}^{2} \hat{\boldsymbol{e}}_{0}(\boldsymbol{K}, \lambda) \hat{\boldsymbol{e}}_{m}(\boldsymbol{K}, \lambda), \tag{48}
\end{equation*}
$$

where

$$
\begin{gather*}
\hat{\boldsymbol{e}}_{m}(\boldsymbol{K}, \lambda=1)=(\sin \phi,-\cos \phi, 0) \\
\hat{\boldsymbol{e}}_{m}(\boldsymbol{K}, \lambda=2)=\left(\cos \theta \cos \phi, \cos \theta \sin \phi,-n_{m}^{-1} \sin \theta\right) \tag{49}
\end{gather*}
$$

In the case of vacuum $\left(n_{m}=1\right), \stackrel{\leftrightarrow}{U}_{m}$ reduces to

$$
\begin{equation*}
\stackrel{U}{U}_{V a c}=\sum_{\lambda=1}^{2} \hat{\boldsymbol{e}}_{0}(\lambda) \hat{\boldsymbol{e}}_{0}(\lambda)=\stackrel{\leftrightarrow}{\boldsymbol{I}}-\frac{\boldsymbol{K} \boldsymbol{K}}{K^{2}} \tag{50}
\end{equation*}
$$

From Eq. (45) we see that the field at $\boldsymbol{r}(x, y, z)$ in the 1 DPC is equal to the field at $\widetilde{\boldsymbol{r}}\left(x, y, \widetilde{z}_{m}(z)\right)$ in the vacuum; that is to say, the field in the 1 DPC of $I-M$ can be determined through the calculation of the vacuum field distribution. The field at $\widetilde{\boldsymbol{r}}$ can be considered as a superposition of the contributions of two perpendicular components of $\boldsymbol{p}$ (see the Appendix); one is

$$
\begin{align*}
\langle 0| \boldsymbol{E}^{(+)}(\boldsymbol{r}, t)\left|\gamma_{0}(t)\right\rangle_{\|}= & 2 n_{m}^{-1} E_{0} \cos \eta \int_{0}^{\infty} d K \frac{e^{-\left(\Gamma_{0} / 2\right) t}-e^{-i c(K-k) t}}{c(K-k)+i \frac{\Gamma_{0}}{2}} \\
& \times e^{-i c k t}\left(\frac{\cos (K \widetilde{r})}{K^{2} \widetilde{r}^{2}}-\frac{\sin (K \widetilde{r})}{K^{3} \widetilde{r}^{3}}\right), \tag{51}
\end{align*}
$$

and another is

$$
\begin{align*}
\langle 0| \boldsymbol{E}^{(+)}(\boldsymbol{r}, t)\left|\gamma_{0}(t)\right\rangle_{\perp}= & E_{0} \sin \eta \int_{0}^{\infty} d K \frac{e^{-\left(\Gamma_{0} / 2\right) t}-e^{-i c(K-k) t}}{c(K-k)+i \frac{\Gamma_{0}}{2}} \\
& \times e^{-i c k t}\left(\frac{\sin (K \widetilde{r})}{K \widetilde{r}}+\frac{\cos (K \widetilde{r})}{K^{2} \widetilde{r}^{2}}-\frac{\sin (K \widetilde{r})}{K^{3} \widetilde{r}^{3}}\right), \tag{52}
\end{align*}
$$

where

$$
\begin{equation*}
E_{0}=\frac{p c k^{3}}{(2 \pi)^{2} \varepsilon_{0}} \tag{53}
\end{equation*}
$$

and $\eta$ is the angle between $\boldsymbol{p}$ and $\widetilde{\boldsymbol{r}}$. The intensity of the field is given by

$$
\begin{equation*}
\left.\left.G^{(1)}(\boldsymbol{r}, \boldsymbol{r} ; t, t)=\left|\langle 0| \boldsymbol{E}^{(+)}(\boldsymbol{r}, t)\right| \gamma(t)\right\rangle\left._{\perp}\right|^{2}+\left|\langle 0| \boldsymbol{E}^{(+)}(\boldsymbol{r}, t)\right| \gamma(t)\right\rangle\left._{\|}\right|^{2} . \tag{54}
\end{equation*}
$$

It should be noted that in the derivation of Eqs. (51) and (52), we have included both outcoming wave $e^{i K r}$ and incoming wave $e^{-i K r}$. The incoming wave $e^{-i K r}$ was neglected in Ref. [14]. The reason for including $e^{-i K r}$ is that in general the reflected wave exists in the multilayer structure. In addition,


FIG. 2. (Color) The space distribution of the radiated field for LHM-RHM multilayers with the parameters shown in the figure. (a) The field in the $y-z$ plane. (b) The field in the $x-z$ plane. The time is $t=1000 / \omega$.
in LHM, the phase increases along the "incoming" direction and the energy flow direction is opposite to the wave vector direction. As a result of including both $e^{i K r}$ and $e^{-i K r}$, the superposition of them leads to the field in an oscillating man-
ner rather than in a monotonous manner decrease away from the atom.

From the expression of the field, Eq. (45), we see that the difference in the LHM-RHM from that in the free space is
that the effective distance $\widetilde{z}_{m}(z)$ can increase or decrease as $z$ increase. This leads the field to change periodically along the $z$-axis direction. At the position of $\widetilde{z}_{m}(z)=0(z \neq 0)$, the field will repeat the distribution as around $z=0$. The field around the atom produces an image in each layer, as shown in Fig. 2, where we give the field distributions in the $y-z$ plane and in the $x-z$ plane, respectively, in (a) and in (b) for $\boldsymbol{p}=p(1,0,0)$ and $z_{a}=0$. In the calculation we have chosen $d_{A / B}=\lambda_{0} / 2, n_{A}=-1$, and $n_{B}=1$, and on both sides of the atom the structure is $(A B)^{3} A$. We only draw the field distribution in the half space on the right side. The field distribution on the left side is symmetrical to the distribution on the right side.

We have seen from Fig. 2 that the radiated field is repeated in turn in each layer. It spreads in the outside space as if the radiation field is emitted from the last image. According to Snell's law, when a light enters LHM from RHM, the light ray is refracted back to the direction of a negative refraction angle. In the above structure each RHM-LHM interface acts as a perfect lens [9,15]. As we know from Ref. [15], the interface of RHM-LHM acts as a perfect lens. If the distance between the source and interface of the first LHM layer is $x_{1}$, then the first focus point in the LHM layer is at the position $2 x_{1}$, far from the source, and the second focus point in the other side is at the position which its distance from the source is $2 d$, where $d$ is the thickness of the LHM layer. In our case, the two-level atom takes the role of a point source and its radiated field will be focused in each layer. From Fig. 2, each layer has the same width and the atom is in the middle of one layer, so all the focus images are in the middle of each layer. Due to the effect of focus in the LHMRHM multilayer, the field is guided to propagate along a small region around the focal points in each layer. If we decrease the thickness of each layer, the intensity of radiation field will be mainly concentrated along the $z$ axis. It looks like the field of the atom is radiated along the $z$ axis without spread. With such a structure, we can control the distribution of the SpE field of the atom without changing the SpE rate.

## V. CONCLUSION

We have constructed a complete set of mode functions for the whole space, including a finite 1DPC and its outsides, as a basis to quantize the radiation field. The dynamic equation of the SpE of a two-level atom is given in formality. The radiated field distribution is calculated for an $I-M$ structure. We find that the propagation of the radiated field is in a directive way. The radiated field is focused in each layer due to negative refraction. The field could be guided to a narrow region by decreasing the thickness of all layers. This structure changes the sphere propagation of the radiation field into a directional propagation which keeping the same SpE rate. It also provides a method to detect the near-field distribution at a far position for the atomic spontaneous emission field similar to the perfect lens.

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FIG. 3. Decomposition of an atomic dipole $\boldsymbol{p}$ into two perpendicular components, one parallel to $\widetilde{\boldsymbol{r}}$ and the other perpendicular to $\widetilde{r}$.
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## APPENDIX

Equation (45) can be rewritten as

$$
\begin{align*}
\langle 0| \boldsymbol{E}^{(+)}(\boldsymbol{r}, t)\left|\gamma_{0}(t)\right\rangle= & i A \int_{0}^{\pi} d \theta \sin \theta \int_{0}^{2 \pi} d \phi \int_{0}^{\infty} d K \int_{0}^{t} d t^{\prime} \\
& \times e^{-i\left(\omega_{0}-\nu_{K}\right) t^{\prime}-\left(\Gamma_{0} / 2\right) t^{\prime}} e^{-i \nu_{K} t} \exp [i \boldsymbol{K} \cdot \widetilde{\boldsymbol{r}}] \boldsymbol{p} \cdot \stackrel{\leftrightarrow}{U} \tag{A1}
\end{align*}
$$

where

$$
\begin{equation*}
A=\frac{V\left(\varepsilon_{K}^{0}\right)^{2} k^{2}}{\hbar(2 \pi)^{3}} . \tag{A2}
\end{equation*}
$$

We have replaced $K^{2}$ by $k^{2}$ and taken it out from the integral according to the Weisskopf-Weigner approximation.

In order to carry out the integral (A1), we rotate the coordinate system such that the vector $\widetilde{\boldsymbol{r}}$ points along the $z^{\prime}$ axis and the atomic dipole moment $\boldsymbol{p}$ forms an angle $\eta$ with the $z^{\prime}$ axis in the $x^{\prime}-z^{\prime}$ plane. The field at $\tilde{\boldsymbol{r}}$ can be consider as a sum of the contributions of two perpendicular components of $\boldsymbol{p}=p(\sin \eta, 0, \cos \eta)$, one parallel to $\widetilde{\boldsymbol{r}}$ and the other perpendicular to $\widetilde{\boldsymbol{r}}$, as shown in Fig. 3. The contribution of the parallel component can be determined by calculating the field of a dipole $p(0,0, \cos \eta)$ along the $z^{\prime}$-axis, and the contribution of the perpendicular component can be determined by calculating the field of a dipole $p(\sin \eta, 0,0)$ along the $z^{\prime}$-axis.

For $\boldsymbol{p}=p(0,0, \cos \eta)$, we find the integrating over $\phi$ leads the $x^{\prime}$ component and the $y^{\prime}$ component of the field to be equal to zero,

$$
\begin{equation*}
\langle 0| \boldsymbol{E}^{(+)}\left(z^{\prime}, t\right)\left|\gamma_{0}(t)\right\rangle_{x^{\prime}}=0, \tag{A3}
\end{equation*}
$$

$$
\begin{equation*}
\langle 0| \boldsymbol{E}^{(+)}\left(z^{\prime}, t\right)\left|\gamma_{0}(t)\right\rangle_{y^{\prime}}=0 . \tag{A4}
\end{equation*}
$$

The $z^{\prime}$ component of the field is

$$
\begin{align*}
\langle 0| \boldsymbol{E}^{(+)}\left(z^{\prime}, t\right)\left|\gamma_{0}(t)\right\rangle_{z^{\prime}} & =i A p \cos \eta \int_{0}^{\pi} d \theta \sin \theta \int_{0}^{2 \pi} d \phi \int_{0}^{\infty} d K \int_{0}^{t} d t^{\prime} e^{-i\left(\omega_{0}-\nu_{K}\right) t^{\prime}-\left(\Gamma_{0} / 2\right) t^{\prime}} e^{-i \nu_{K^{t}}} \exp \left[i K z^{\prime} \cos \theta\right]\left(-n_{m}^{-1}\right)\left(1-\cos ^{2} \theta\right) \\
& =-i 2 \pi A p n_{m}^{-1} \cos \eta \int_{0}^{\infty} d K \frac{e^{-i c(k-K) t-\left(\Gamma_{0} / 2\right) t}-1}{-i c(k-K)-\frac{\Gamma_{0}}{2}} e^{-i c K t} \int_{-1}^{1} d u\left(1-u^{2}\right) \exp \left[i K z^{\prime} u\right] \\
& =-i 4 \pi A p n_{m}^{-1} \cos \eta \int_{0}^{\infty} d K \frac{e^{-i c(k-K) t-\left(\Gamma_{0} / 2\right) t}-1}{-i c(k-K)-\frac{\Gamma_{0}}{2}} e^{-i c K t}\left(-\frac{e^{i K z^{\prime}}+e^{-i K z^{\prime}}}{K^{2} z^{\prime 2}}+\frac{e^{i K z^{\prime}}-e^{-i K z^{\prime}}}{i K^{3} z^{\prime 3}}\right) \\
& =2 n_{m}^{-1} E_{0} \cos \eta \int_{0}^{\infty} d K \frac{e^{-\left(\Gamma_{0} / 2\right) t}-e^{-i c(K-k) t}}{c(K-k)+i \frac{\Gamma_{0}}{2}} e^{-i c k t}\left(\frac{\cos \left(K z^{\prime}\right)}{K^{2} z^{\prime 2}}-\frac{\sin \left(K z^{\prime}\right)}{K^{3} z^{\prime 3}}\right) \tag{A5}
\end{align*}
$$

where $E_{0}=4 \pi A p=p c k^{3} /(2 \pi)^{2} \varepsilon_{0}$. Recalled that $z^{\prime}=\widetilde{r}$; we have

$$
\begin{equation*}
\langle 0| \boldsymbol{E}^{(+)}(\boldsymbol{r}, t)\left|\gamma_{0}(t)\right\rangle_{\|}=2 n_{m}^{-1} E_{0} \cos \eta \int_{0}^{\infty} d K \frac{e^{-\left(\Gamma_{0} / 2\right) t}-e^{-i c(K-k) t}}{c(K-k)+i \frac{\Gamma_{0}}{2}} e^{-i c k t}\left(\frac{\cos (K \widetilde{r})}{K^{2} \widetilde{r}^{2}}-\frac{\sin (K \widetilde{r})}{K^{3} \widetilde{r}^{3}}\right), \tag{A6}
\end{equation*}
$$

where the subscript $m$ is determined by the layer where $\boldsymbol{r}$ is located.
For $\boldsymbol{p}=p(\sin \eta, 0,0)$, we find the integrating over $\phi$ leads the $y^{\prime}$ component and the $z^{\prime}$ component of the field to be equal to zero,

$$
\begin{align*}
& \langle 0| \boldsymbol{E}^{(+)}\left(z^{\prime}, t\right)\left|\gamma_{0}(t)\right\rangle_{y^{\prime}}=0,  \tag{A7}\\
& \langle 0| \boldsymbol{E}^{(+)}\left(z^{\prime}, t\right)\left|\gamma_{0}(t)\right\rangle_{z^{\prime}}=0 . \tag{A8}
\end{align*}
$$

The $x^{\prime}$ component

$$
\begin{align*}
\langle 0| \boldsymbol{E}^{(+)}\left(z^{\prime}, t\right)\left|\gamma_{0}(t)\right\rangle_{x^{\prime}} & =i A p \sin \eta \int_{0}^{\pi} d \theta \sin \theta \int_{0}^{2 \pi} d \phi \int_{0}^{\infty} d K \int_{0}^{t} d t^{\prime} e^{-i\left(\omega_{0}-\nu_{K}\right) t^{\prime}-\left(\Gamma_{0} / 2\right) t^{\prime}} e^{-i \nu_{K} t} \exp \left[i K z^{\prime} \cos \theta\right]\left[1-\sin ^{2} \theta \cos ^{2} \phi\right] \\
& =i \pi A p \sin \eta \int_{0}^{\infty} d K \frac{e^{-i c(k-K) t-\left(\Gamma_{0} / 2\right) t}-1}{-i c(k-K)-\frac{\Gamma_{0}}{2}} e^{-i c K t} \int_{-1}^{1} d u\left(1+u^{2}\right) \exp \left[i K z^{\prime} u\right] \\
& =i 2 \pi A p \sin \eta \int_{0}^{\infty} d K \frac{e^{-i c(k-K) t-\left(\Gamma_{0} / 2\right) t}-1}{-i c(k-K)-\frac{\Gamma_{0}}{2}} e^{-i c K t}\left(\frac{e^{i K z^{\prime}}-e^{-i K z^{\prime}}}{i K z^{\prime}}+\frac{e^{i K z^{\prime}}+e^{-i K z^{\prime}}}{K^{2} z^{\prime 2}}-\frac{e^{i K z^{\prime}}-e^{-i K z^{\prime}}}{i K^{3} z^{\prime 3}}\right) \\
& \left.=E_{0} \sin \eta \int_{0}^{\infty} d K \frac{e^{-\left(\Gamma_{0} / 2\right) t}-e^{-i c(K-k) t}}{c(K-k)+i \frac{\Gamma_{0}}{2}} e^{-i c k t( } \frac{\sin \left(K z^{\prime}\right)}{K z^{\prime}}+\frac{\cos \left(K z^{\prime}\right)}{K^{2} z^{\prime 2}}-\frac{\sin \left(K z^{\prime}\right)}{K^{3} z^{\prime 3}}\right) \tag{A9}
\end{align*}
$$

So we get

$$
\begin{equation*}
\langle 0| \boldsymbol{E}^{(+)}(\boldsymbol{r}, t)\left|\gamma_{0}(t)\right\rangle_{\perp}=E_{0} \sin \eta \int_{0}^{\infty} d K \frac{e^{-\left(\Gamma_{0} / 2\right) t}-e^{-i c(K-k) t}}{c(K-k)+i \frac{\Gamma_{0}}{2}} e^{-i c k t}\left(\frac{\sin (K \widetilde{r})}{K \widetilde{r}}+\frac{\cos (K \widetilde{r})}{K^{2} \widetilde{r}^{2}}-\frac{\sin (K \widetilde{r})}{K^{3} \widetilde{r}^{3}}\right) \tag{A10}
\end{equation*}
$$

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